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DYNAMIC ANALYSIS OF FUNCTIONALLY GRADED SHELL STRUCTURE USING FINITE ELEMENT METHOD

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ABSTRACT

The present paper deals with the development of a finite element procedure to carry out the dynamic analysis of functionally graded (FG) shell structures. Eight noded shell elements implementing Koitert shell theory and using Mindline's hypothesis having five degrees of freedom per node have been used. Newmark integration scheme has been used for dynamic analysis of FG shell structures. The developed code has been validated with already published results and has been used to study the effect of important parameters on the dynamic response of FG shell structures. It has been observed that the increase in power law exponent as well as radius of curvature, lead to the increase in central deflection of cylindrical FG shell structures.

Keywords: Functionally Graded Materials, Finite Element, Shell Structures.

1. INTRODUCTION

Composite materials are widely used in aerospace application due to their high specific strength and high specific stiffness. But presence of the inter-laminar stresses, residual stresses, de-bonding causes failure of structure and become even more severe in high thermal loading condition. A new kind of material known as functionally graded material (FGM) is developed to overcome these difficulties. In FGM there is gradual change in composition (in thickness direction only) from heat resistant ceramic to fracture resistant metal and also the gradual variation in volume fraction of the constituents.

Praveen and Reddy [1] studied the thermo-elastic behaviour of functionally graded plates by considering first-order shear deformation theory (FSDT) and von Karman nonlinearity for mathematical modelling. Reddy [2] proposed theoretical and finite element model of FG plates based on third-order shear deformation theory (TSDT) and investigated the dynamic behaviour of FG plates. Based on classical small deflection plate theory free vibration analysis and transient response of initially stressed functionally graded thin plate due to impulsive patch load are computed by Yang and Shen [3]. Three dimensional exact solution for vibration of Al/ZrO2 FG rectangular plates were obtained by Vel and Batra [4] and found that the solutions were close to that of FSDT and TSDT. Matsunaga [5] analysed the natural frequencies and buckling stresses of FGM shallow shells by considering the effect of transverse shear and normal deformations, and rotary inertia.

A number of successful attempts are reported in the literatures for the static and dynamic analysis of FGM plates subjected to mechanical loading, but the forced response analysis of FGM shells are not found in any literature. In the present work finite element modeling of doubly-curved FG shell based on Koiter's shell theory with deformation following Mindlin's hypothesis using an eight noded element with five degree of freedom per node has been accomplished. To carry out forced response analysis Newmark direct integration scheme has been used.

2. THEORITICAL FORMULATION

2.1. Modeling of FGM properties

For the present study, the functionally graded material of thickness h, consisting of two constituent materials has been considered. The effective material properties of FGM shell is obtained by power law distribution, and can be expressed as

$$P(z) = (P_t - P_b)\left(\frac{2z+h}{2h}\right)^{\lambda} + P_b \qquad (1)$$

where P denotes the material property at any location z, P_t and P_b are the material properties of the top surface (ceramic) and bottom surface (metal) respectively. Volume fraction exponent $\lambda = 0$ represents a component fully made of ceramic, while the content of metal increases as λ increases. Poisson's ratio, v is assumed to be constant throughout the material. For the FGMs the stress-strain relationship can be expressed as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \frac{E(z)}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0\\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0\\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0\\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \quad (2)$$

2.2 Finite Element Formulation

As shown in Figure 1, an orthogonal curvilinear coordinate system (α_1, α_2) with the mid-plane of the shell assumed to be the reference surface is employed in the analysis. In the present paper finite element formulation of FGM shell is based on stress-resultant type Koiter's shell theory [6] in which effect of shear deformation is considered.

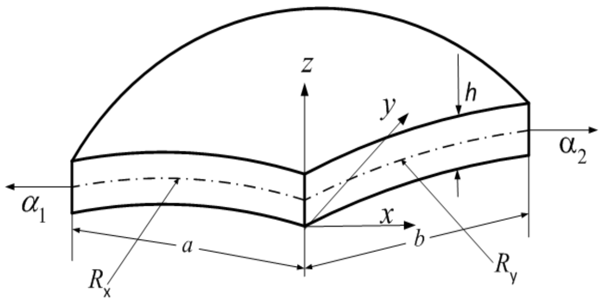

Fig 1. Doubly curved FG shell

The shell mid-surface in the rectangular cartesian coordinate system (x, y, z) mapped into the parametric space (α_1, α_2) has been divided into required number of quadrilateral elements or sub-domains. Isoparametric mapping concept is used to map the quadrilateral element in the curvilinear coordinates (α_1, α_2) into the natural coordinates.

The curvilinear coordinates (α_1, α_2) of any point within an element may be expressed as

$$\alpha_1 = \sum_{i=1}^{nd} N_i(\xi, \eta)\,\alpha_{1i} \qquad \alpha_2 = \sum_{i=1}^{nd} N_i(\xi, \eta)\,\alpha_{2i} \quad (3)$$

where α_{1i}, α_{2i} are the curvilinear co-ordinates of the i^{th} node and nd is the number of nodes in an element and N_i is the shape function corresponding to i^{th} the node. The displacement components on the shell mid-surface at any point within an element may be expressed as

$$\begin{Bmatrix}u_0\\ v_0\\ w\\ \theta_1\\ \theta_2\end{Bmatrix} = \sum_{i=1}^{nd} N_i \begin{Bmatrix}u_{0i}\\ v_{0i}\\ w_i\\ \theta_{1i}\\ \theta_{2i}\end{Bmatrix} \quad (4)$$

where u_{0i}, v_{0i} and w_i are the deflection of mid-surface at i^{th} node in α_1, α_2 and z directions, respectively. θ_{1i} is the rotation of normal at i^{th} node about α_2-axis and θ_{2i} is the rotation of normal at i^{th} node about α_1-axis.

In the present work normal strain component is neglected in the thickness direction, therefore the five strain components of a doubly curved shell can be written as

$$\{\varepsilon_{11} \;\; \varepsilon_{22} \;\; \varepsilon_{12} \;\; \varepsilon_{23} \;\; \varepsilon_{13}\}^T = \{\varepsilon_{11}^0 \;\; \varepsilon_{22}^0 \;\; \varepsilon_{12}^0 \;\; \varepsilon_{23}^0 \;\; \varepsilon_{13}^0\}^T + z\{k_{11} \;\; k_{22} \;\; k_{12} \;\; 0 \;\; 0\} \quad (5)$$

where ε_{11}^0, ε_{22}^0, ε_{12}^0, ε_{23}^0, ε_{13}^0 correspond to mid-surface strains, and k_{11}, k_{22} and k_{12} correspond to curvatures of mid-surface.

By using eight noded isoparametric shape functions, the strain-displacement relationship can be expressed as

$$\{\varepsilon\} = [B]\{d\} \quad (6)$$

where $[B]$ is the strain –displacement matrix.

In this work finite element equations are obtained by employing the extended Hamilton's principle as

$$\int_{t_1}^{t_2} (\delta L^e + \delta W^e)\, dt = 0 \quad (7)$$

where the Lagrangian $L^e = T^e - U^e$, T^e is the kinetic energy, $U^e = U_m^e$ the internal potential energy consists of the elastic strain energy of the entire structure, and δW^e is the virtual work of external forces.

After substituting all the energy expressions in Eq. (3) and arranging it, the following equation is obtained

$$[M_{uu}^e]\{\ddot{d}^e\} + [K_{uu}^e]\{d^e\} = \{F_v^e\} + \{F^e\} + \{F_p^e\} + \{F_T^e\} \quad (8)$$

where the matrices for the element are defined as

Mass matrix

$$[M_{uu}^e] = \int_{v^e} [N_u]^T [\rho^e] [N_u]\, dV \quad (9)$$

Structural stiffness matrix

$$[K_{uu}^e] = \int_{v^e} [B_u]^T [c] [B_u]\, dV \quad (10)$$

Load vector due to body applied force

$$\{F_v^e\} = \int_{v^e} [N_u]^T \{F_v(x,y,z)\}\, dV \qquad (11)$$

Load vector due to distributed load

$$\{F^e\} = \int_{\Omega_1^e} [N_u]^T \{f^e(x,y)\}\, dA \qquad (12)$$

Load vector due to point loads

$$\{F_p^e\} = [N_u]^T \{F_p\} \qquad (13)$$

3. RESULTS AND DISCUSSIONS

The developed code is first validated with the already published results before using it for generating new results. Since no literature has been reported for the forced response of FG shell, therefore results from the present code is validated with already published results corresponding to plate.

3.1 Validation

To check the accuracy of the present code the dynamic response of simply supported square isotropic plate subjected to suddenly applied uniform load q = 100 kN/m^2 is determined and compared with the already published result of Kant et al. [7] as shown in Fig. 3. The dimensions and material properties of the plate are $a = b$ = 250 mm, h = 50 mm, E = 21 GPa, ν = 0.25, ρ = 800 kg/m^3. It is observed from Fig. 3 that the result from the present code converges well with 8×8 mesh size and in excellent agreement with the already published results of Kant et al. [7].

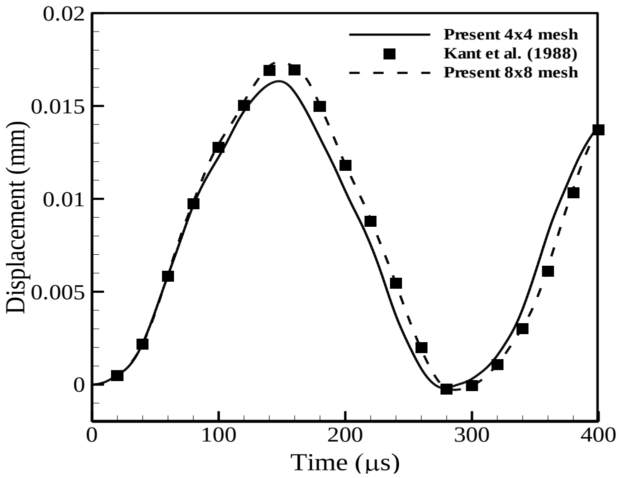

Fig 2. Variation of central deflection with time

3.2 Forced Response Analysis

The developed finite element code has been validated with the existing results and some important results are generated for forced response analysis of FG cylindrical shells. A simply supported Al_2O_3/Al FGM cylindrical shell subjected to a uniformly distributed load q of 10 kN/m^2 and a time step Δt = 10 μs is considered for the forced response analysis. The thickness, side and radius of curvature of the FGM shell are h= 0.1 m, $a = b$ = 1 m and R_x =10,000, R_y = 1 m respectively. Material properties of Aluminum (Al) and Alumina (Al_2O_3) are Al: E_m = 70 GPa, ν_m = 0.3, ρ_m = 2702 kg/m^3 Al_2O_3: E_m = 380 GPa, ν_m = 0.3, ρ_m = 3800 kg/m^3 respectively.

Figure 1 shows the effect of power law gradient on the central deflection of the simply supported Al_2O_3/Al FG cylindrical shell. From Fig. 1 it could be seen that power law gradient strongly affect the central deflection of FG shells and as the power law gradient increases the central deflection also increases due to reduced stiffness.

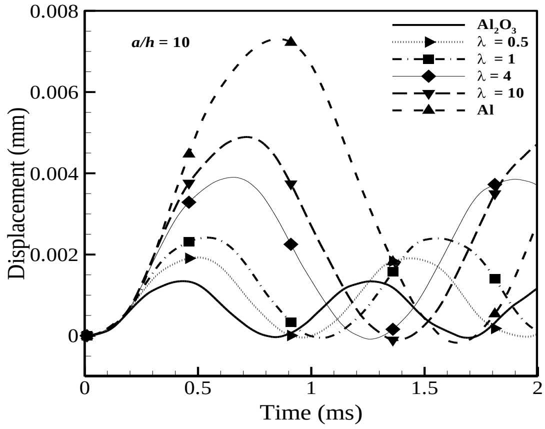

Fig 3. Effect of power law gradient on central deflection of Al_2O_3/Al FG cylindrical shell

Effect of radius of curvature on central deflection of FG cylindrical shell is shown in figure 4. It has been observed that as the radius of curvature increases central deflection also increases and after attaining some value of radius of curvature there is not much change in the central deflection due to the fact that the FG cylindrical shell is attaining the shape of the plate.

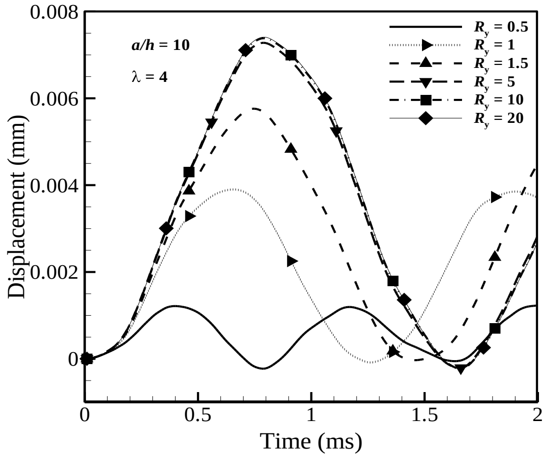

Fig 4. Effect of radius of curvature on central deflection of Al_2O_3/Al FG cylindrical shell

4. CONCLUSIONS

In the present work a finite element codel has been developed to carry out dynamic analysis of FG shells. The developed code has been used to study the effect of some of the important parameters in the forced response of a FG cylindrical shell. It has been noticed that with the increase in the power law exponent value, central deflection of the shell also increases. Effect of radius of curvature on the central deflection of FG shell is also studied and it is observed that increase in the radius of curvature increases central deflection of FG shells.

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